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RESEARCH

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Frequency spectrum of engineering structures with time varying masses

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Abstract

Time varying mass engineering structure is the type of structure where the total mass of the structure changes rapidly, and the loads generated are dependent on the properties of the structure, discharge rate and the properties of the materials inside the structure. A discharging silo is a typical time varying mass civil engineering structure where the total mass of the entire structure changes during the discharge of the stored materials. This paper presents a novel combinatory approach using Hilbert Transform and Variational Mode Decomposition methods to analyse nonlinear and non-stationary signals collected from the experimental silo filled with iron ore and one filled with sand. In particular, the silo filled with sand was producing loud foghorn like sound during discharge. The results revealed the existence of a frequency spectrum that can be developed further to assess the structural integrity of a time varying mass civil engineering structure such as a silo without the impacting silo operation and throughput. The obtained frequency spectrum shows that there is a noticeable difference between the spectra for the silo filled with iron ore and the silo filled with sand.

Keywords: HHT, HMS, VMF, VMD, Frequency spectrum, Time varying mass civil engineering structure

Introduction

Time varying mass engineering structures refer to civil engineering structures where the total mass of the structure changes with time. The most important characteristic of a time varying mass engineering structure is the existence of a dynamic load associated with the sudden reduction in the total mass of the structure. The magnitude of the dynamic load generated due to the sudden reduction in the total mass of the structure is governed by the initial conditions of the structure, such as elastic deformations of the overall structure and induced vibrations from plants and machinery operating nearby; dynamic properties of the overall structure and foundation such as mass, damping and stiffness; and engineering properties of the content carried by the structure. In fact, this mass change phenomenon occurs in most civil engineering structures such as bridges, cranes, ships, buildings, stadiums, liquid storage tanks and silos. However, in most civil engineering structures, the dynamic load generated by the sudden reduction in the total mass of the structure is often negligible and can be adequately accounted for by applying a dynamic factor onto the equivalent static load because of the slow rate of change of the total mass. The practice of neglecting the dynamic load generated by the sudden reduction of the total mass of the structure may lead to overloading and fatigue failure in a storage structure such as a silo. In such storage structure,it is beneficial operationally to retrieve the stored content as quickly as possible and is used frequently to maintain or improve production throughput. It is important to note that the dynamic load generated from the sudden unloading can cause the content inside the structure in motion to experience g-forces and the content can bounce around inside the structure and various other phenomena.

Accelerometers are devices designed to measure the accelerations of a moving or vibrating body. Such devices are commonly used in various field of engineering for

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example earthquake, structural health monitoring and vibrating machinery. The acceleration signals are often processed using a traditional algorithm, such as Fast Fourier Transform (FFT), which is an algorithm to transform signals from time domain to frequency domain [1]. As mentioned by He and Yi [2], FFT is more suited to stationary signal processing and feature extraction. The effectiveness of FFT algorithm decreases when processing nonlinear and non-stationary problems [2]. Furthermore, according to Huang and Shen [3], the data generated by most natural processes is likely to be both nonlinear and non-stationary.

In signal processing, a signal is linear if it has two mathematical properties such as homogeneity and additivity [4, 5]. Stationary signals' frequencies and spectral contents do not change with respect to time [6] and can be written as discrete sum of cosine waves or exponentials [7]. A signal is classified as non-stationary if the statistical structure of the signal such as mean, variance, correlation or covariance changes as a function of time or do not exhibit stationary features [8].

Prior to the development of Hilbert Huang Transform (HHT), available signal processing methods were more suited to analyse either nonlinear or non-stationary signals but not both. HHT was created to address the needs to analyse nonlinear and non-stationary signals [9] and since has been adopted in many research areas [3]. The HHT algorithm consists of two parts, Empirical Mode Decomposition (EMD) and Hilbert Transform (HT). EMD is based on the assumption that any data consists of simple intrinsic modes of oscillations where each intrinsic mode represents a simple oscillation. The oscillation is assumed to be symmetric with respect to the local mean and different modes of oscillation superimposed to form the final complicated data. Each mode of oscillation is represented by an intrinsic mode function (IMF). An IMF has to satisfy the following conditions:

1. The number of extrema and the number of zero-crossings must either equal or differ at most by one.
2. The mean value of the envelopes defined by the local maxima and the local minima is zero.

The first condition is similar to the narrowband requirements for stationary Gaussian process whereas the second requirement was invented to that the instantaneous frequency will not have unwanted fluctuations induced by asymmetric waveforms. At the end of the sifting process, the original data $x(t)$ is decomposed into a set of IMFs (c_j) and a residual (r_N). HT is applied to the IMFs obtained from the EMD algorithm. According to Feldman [10], unlike other integral transforms such as Fourier and Laplace, HT is not a transform between domains. In HT, the amplitudes of the signal remain unchanged, the phases of the signal are shifted by a quarter of a period. For example, the Hilbert Transform of $cos(\omega t)$, where $\omega > 0$, is $cos\left(\omega t - \frac{\pi}{2}\right)$. As such HT provides a mechanism to determine the instantaneous frequency and amplitude of a signal thus giving better insights into signals generated by time varying mass dynamic system like a silo. HMS represents a measure of the total amplitude or energy contribution from each frequency [3]. There have been two important improvements since the creation of the EMD algorithm, Ensemble Empirical Mode Decomposition (EEMD) and Complete Ensemble EMD with Adaptive Noise (CEEMDAN). Both EEMD and CEEMDAN were introduced to improve the mode extraction ability of the original EMD. However, all the variants of the original EMD method and the EMD method still lack a mathematical foundation. A noticeable development in this area is the Variational Mode Decomposition (VMD) method introduced by Dragomiretskiy and Zosso [11] to replace the traditional EMD method.

A discharging silo is a time varying mass civil engineering structure. During discharge, the total mass of the silo structure changes rapidly, thus affecting the overall dynamic response characteristics of a silo structure [12]. Tu and Vimonsatit [12] classified a discharging silo as a time varying mass structural dynamic problem and provided a numerical equation to describe the granular-structure interaction [13]. To our knowledge, the frequencies of the discharging silo have not been studied in detail to date, primarily due to the unrealistic assumptions in Janssen's silo theory [14].

In this paper, the VMD algorithm is used to decompose acceleration signals generated by the flowing granules from an experimental silo filled with iron ore and one filled with sand into a series of Variation Mode Functions (VMF). The VMFs are transformed using HT method and converted into frequency spectra using the Hilbert Marginal Spectrum (HMS) algorithm. Each discharge cycle produces vibrations of varying amplitudes and frequencies presenting the needs to study such vibrations so the HMS spectrum can be further developed, incorporating influencing parameters, to monitor the structural integrity of the silo in real time negating the need to shutdown the operation of the silo for periodic structural inspection. It is envisaged that significant variations of the frequency spectrum mean the silo structure is deteriorating structurally and warrant structural strengthening.

Hilbert Huang Transform (HHT)

The analysis methods prior to the development of HHT were either linear but non-stationary or nonlinear but stationary and statistically deterministic [9]. However, the data from real world processes are nonlinear and non-stationary. Thus HHT was developed to address the need to analyse nonlinear and non-stationary signals that occur in natural processes. The HHT algorithm consists of two parts, EMD and HT. Since its introduction, EMD has been improved by Ensemble Empirical Mode Decomposition (EEMD) and Complete Ensemble EMD with Adaptive Noise (CEEMDAN). Dragomiretskiy and Zosso [11] introduced Variation Mode Decomposition (VMD) method to replace EMD method.

Variational Mode Decomposition (VMD)

The goal of VMD is to decompose a signal f into a discrete number of sub-signals u_k that have specific sparsity properties while reproducing the input [11]. Dragomiretskiy and Zosso [11] assumed that each mode k to be mostly compact around a center pulsation ω_k, which is determined along with the decomposition. The bandwidth of each mode is assessed by:

1. Compute the associated analytic signal by means of the Hilbert Transform in order to obtain a unilateral frequency spectrum for each mode u_k;
2. Shift each mode's frequency spectrum to 'baseband' by mixing with an exponential tuned to the respective estimated center frequency; and
3. The bandwidth is estimated through the H^1 Gaussian smoothness of the demodulated signal (the squared L^2-norm of the gradient) resulting in a constrained variational problem shown in Eq. 1.

$$\min_{\{u_k\},\{\omega_k\}} \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\} \text{s.t.} \sum_k u_k = f \tag{1}$$

where $\{u_k\} := \{u_1, \ldots, u_K\}$ and $\{\omega_k\} := \{\omega_k, \ldots, \omega_K\}$ are shorthand notations for the set of all models and their center frequencies, respectively. Equally, $\sum_k := \sum_k^K$ is understood to be the summation of all the modes together. Quadratic penalty terms and Lagrangian multipliers, λ, are introduced to remove reconstruction constraints. The quadratic penalty is a classic way to encourage reconstruction fidelity, typically in the presence of additive Independent and Identically Distributed (iid) Gaussian noise. The weight of the penalty term is derived from an inversely proportional Bayesian prior to the noise level in the data. Lagrangian multipliers on the other hand are commonly added to enforce strict constraints. Combining quadratic terms and Lagrangian multipliers results in nice convergence properties of the quadratic penalty at finite weight, and the strict enforcement of the constraint by the Lagrangian multiplier. The complete algorithm is as follows:

1. Initialise $\{\hat{u}_k^1\}, \{\omega_k^1\}, \hat{\lambda}^1, n \leftarrow 0$;
2. Update the values of according to the following formulae:

$$\hat{u}_k^{n+1} \leftarrow \frac{\hat{f}(\omega) - \sum_{i<k} \hat{u}_i^{n+1}(\omega) - \sum_{i>k} \hat{u}_i^n(\omega) + \frac{\hat{\lambda}^n(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k^n)^2} \tag{2}$$

$$\omega_k^{n+1} \leftarrow \frac{\int_0^{+\infty} \omega \left| \hat{u}_k^{n+1}(\omega) \right|^2 d\omega}{\int_0^{+\infty} \left| \hat{u}_k^{n+1}(\omega) \right|^2 d\omega} \tag{3}$$

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^n(\omega) + \tau \left[\hat{f}(\omega) - \sum_k \hat{u}_k^{n+1}(\omega) \right] \tag{4}$$

3. Repeat 2 until the convergence criterion is satisfied (Eq. 5).

$$\sum_k \frac{\| \hat{u}_k^{n+1} - \hat{u}_k^n \|_2^2}{\| \hat{u}_k^n \|_2^2} < \epsilon \tag{5}$$

It was noted by Isham et al. [15] that the performance of VMD is dependent on predetermined number of modes. An inaccurate number of modes will result in over and under decomposition that will affect the Variation Mode Functions (VMFs).

Hilbert transform

HT was introduced by David Hilbert to provide a mechanism to determine the instantaneous frequency and amplitude of a signal. In HHT, HT is applied to the IMFs obtained from the EMD algorithm or VMFs from the VMD algorithm.

The Hilbert Transform $c_H(t)$ of any signal $c(s)$ is defined as:

$$c_H(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c(s)}{t - s} ds \tag{6}$$

$$z(t) = c(s) + jc_H(t) = a(t) e^{j\varnothing(t)} \tag{7}$$

$$a(t) = \sqrt{c(s)^2 + c_H(t)^2} \tag{8}$$

$$\varnothing(t) = \tan^{-1} \frac{c_H(t)}{c(s)} \tag{9}$$

$$w(t) = \frac{d\varnothing(t)}{dt} \tag{10}$$

where:

Fig. 1 Silo containing iron ore after discharge

- P is the Cauchy principal value of the singular integral from which the analytical signal of the *c(s)*.
- *c(s)* is the signal.
- $c_H(t)$ is the Hilbert transform of the original signal.
- *a(t)* is the amplitude of pre-envelope.
- $\phi(t)$ is the instantaneous phase.

Experimental setup

Iron ore silo

In total, there were 34 flow experiments conducted on four different hopper opening sizes, 200 mm, 250 mm, 300 mm and 400 mm, for the silo containing iron ore built at Curtin University. The silo structure shown in Fig. 1 was wired with a total of 16 accelerometers, eight attached to the silo (as shown in Fig. 2) and eight fastened on the supporting structure (Figs. 3 and 4) to record the responses of the silo structure as the granules exit the silo. The serial numbers, data logger channels and locations of the accelerometers on the silo structure are shown in Table 1. However, for the purpose of this paper, only accelerometer data from an experiment conducted on the 300 mm opening hopper were selected to demonstrate how VMD and HHT can be used to decompose signals generated by the flowing granules. The full report of all the experiments was published by Tu [16]. The granular material used in this instance was iron ore donated by Rio Tinto.

The accelerometers installed were seismic ceramic flexural ICP accelerometer model 393B04 made by PCB. They were capable of measuring frequency between 0.06 Hz to 450 Hz and acceleration up to ± 5 g. The accelerometers were chosen based on their ability to sense frequencies within the expected frequency spectrum generated by the silo during discharge. The accelerometers were connected to a data logger model NI9234,

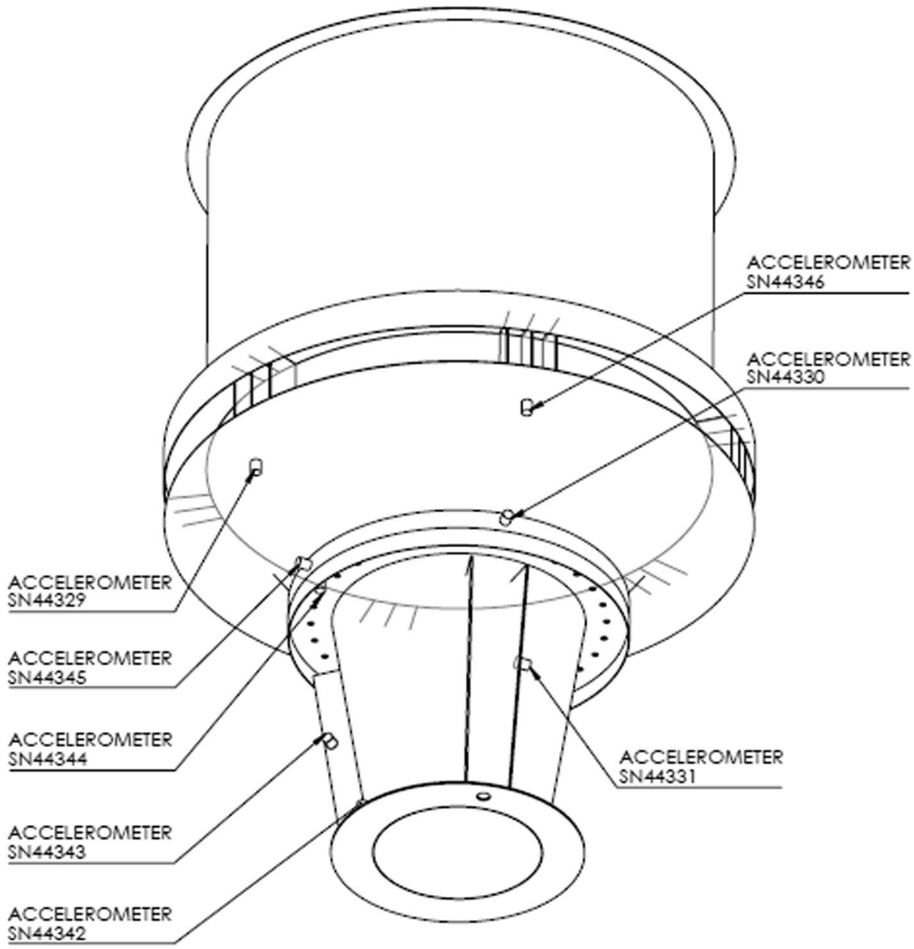

Fig. 2 Accelerometers mounted on the hopper

associated software made by National Instruments, and the sampling rate was set at 2048 samples per second.

The ore was stored in drums and poured in from the top using a forklift and drum lifter. Four bulker bags with spouts at the bottom were placed inside the container at the bottom of the silo to capture and enable the ore to be lifted out of the container by a forklift. The ore inside the bulker bag was lifted out of the container, and the spout at the bottom was untied to allow the ore to flow back into the empty drums. The discharge process was enabled by swinging the lever to unlock the gate.

Sand silo

The experiments conducted by Griffiths [17] were purposely designed to reproduce the honking phenomenon. In Griffiths' experiments, a polycarbonate hollow tube of 2000 mm in length, 194 mm internal diameter and 3 mm wall thickness was used. In total, eight accelerometers (PCB Model 393B04) were screwed to the outside of the silo wall at 200 mm spacings starting 100 mm from the top of the silo (Fig. 5). The accelerometers have a measurement range of plus or minus ± 49 m/s^2 and a frequency range between 0.06 and 450 Hz. All accelerometers were connected to a QuantumX MX840B data logger, and the sampling rate of 1200 Hz was adopted.

Sand with a grain diameter of 300–600 μm was chosen to be the fill material for the experiment because it has repeatedly and reliably induced the vibration and booming sound. The sand particle size of 300–600 μm was selected as it fell within the range of most previous studies.

The sand was dried and sieved utilising the multilevel sieve following the procedures set out in AS 1289.3.6.1 2009 [18]. However, only 300–600 μm sand particles were retained. The sand was thoroughly washed in distilled water until sufficient clarity was observed in the wash water to minimise the effect of contaminants in the material. A manageable amount of sand was poured into a bin and filled with water. The sand was then agitated by hand to pull any powders and impurities into the water

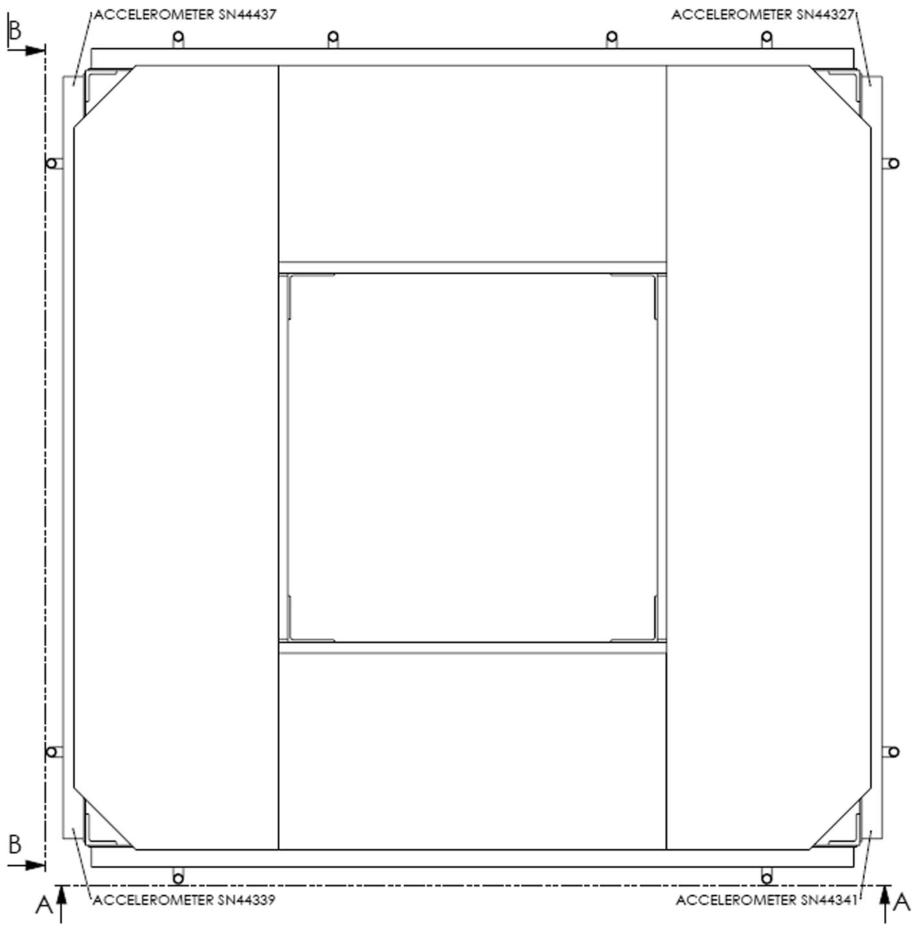

Fig. 3 Accelerometer mounted on the frame – vertical movements

column. The dirty water was then removed and refilled with clean water. This process was repeated until sufficient clarity in the agitated water column was achieved. It was impractical to place all 100 kg of sand in the oven, thus, the cleaned sand was spread in drying trays and allowed to dry indoors for five days. Griffiths [17] varied the moisture content of the sand and conducted multiple experiments. Only one set of data collected on 2 October 2018 is selected for the purposes of this paper.

Results and discussion

Once the silos were filled with ore, the gate at the bottom of the silo was opened to discharge the ore into the tray below. The acceleration responses were decomposed with VMD, outlined in Hilbert Huang Transform (HHT), into their underlying VMFs and transformed using HT as shown in Eq. 6 to obtain the instantaneous frequencies and amplitudes at the locations with accelerometers installed. The VMD and HT algorithms were provided by Matlab 2022a edition.

Figures 6 to 8 illustrate the frequencies of the signals generated by the flowing granules from the silos filled with iron ore and sand, respectively. The total amplitudes shown in Figs. 6 and 7 demonstrate that a discharging silo (shown in Figs. 2 to 4) generates multiple frequencies and the dominant frequencies are predominantly below 20 Hz. Accelerometers (SN44437 and SN44327) mounted on the support frame to measure vertical accelerations reported noticeable dominating frequencies of approximately 260 Hz, 425 Hz, and 450 Hz. Vibrations at frequencies between 20 and 20000 Hz can cause loud sounds to be heard if the amplitudes are sufficiently high. However, the total amplitudes at these frequencies were sufficiently low and no booming sound was heard during the experiments, thus, having negligible effects. A similar conclusion could be adopted for accelerometer SN44328 mounted on the frame to measure lateral accelerations.

Accelerometer SN44344 mounted on the transition region between the two hoppers (shown in Fig. 2) to measure vertical accelerations reported noticeable vibrations. There were multiple frequencies decoded by

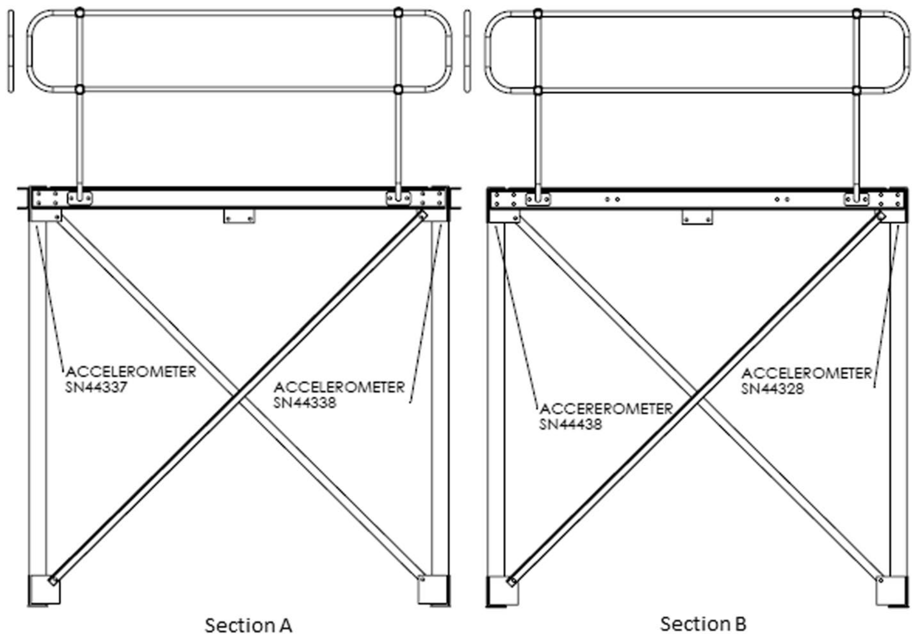

Fig. 4 Accelerometer mounted on the frame – lateral movements

Table 1 Accelerometer serial numbers and datalogger channels

Accelerometer	Serial Number	Dataloger Channel	Accelerometer	Serial Number	Dataloger Channel
1	SN44339	L1C0	9	SN44337	L3C0
2	SN44437	L1C1	10	SN44338	L3C1
3	SN44345	L1C2	11	SN44330	L3C2
4	SN44344	L1C3	12	SN44343	L3C3
5	SN44342	L2C0	13	SN44331	L4C0
6	SN44328	L2C1	14	SN44346	L4C1
7	SN44341	L2C2	15	SN44327	L4C2
8	SN44329	L2C3	16	SN44438	L4C3

VMD and marked by a number of spikes between 20 and 350 Hz. The total amplitudes of those spikes inform that the silo at that location was vibrating vertically and there were a number of frequencies that needed to be interpreted by the engineer.

In Fig. 8, the accelerometers placed near the top of the sand silo recorded large total amplitudes and frequencies compared to the ones near the bottom. Distinct frequencies can be observed in Fig. 8 coupled with loud sounds being heard by bystanders suggest the phenomenon called silo honking was present. It is interesting to note that accelerometers 4 to 8 recorded lower total amplitudes within the effective frequency range of the accelerometers compared to the ones near the top of the sand silo. Structurally, the radial stiffness of the cylinder near the top is much lower than at the middle of the cylinder and near the bottom where the support is present. The amplitude of vibration is inversely proportional to the stiffness of the structure unless resonance is encountered. The total amplitudes for accelerometer 8 demonstrate that the amplitude is significantly reduced near the structural support due to the increased radial stiffness. This suggests that the amplitudes of vibration near the top of the silo can be reduced by adding stiffening rings to increase the radial stiffness of the silo. A trend that can be reported after studying all the frequency spectra is that, the stiffer the structure the less the amplitude of vibration unless resonance is encountered. In the authors' opinion, resonance is difficult to achieve but not impossible in a time varying mass engineering structure because

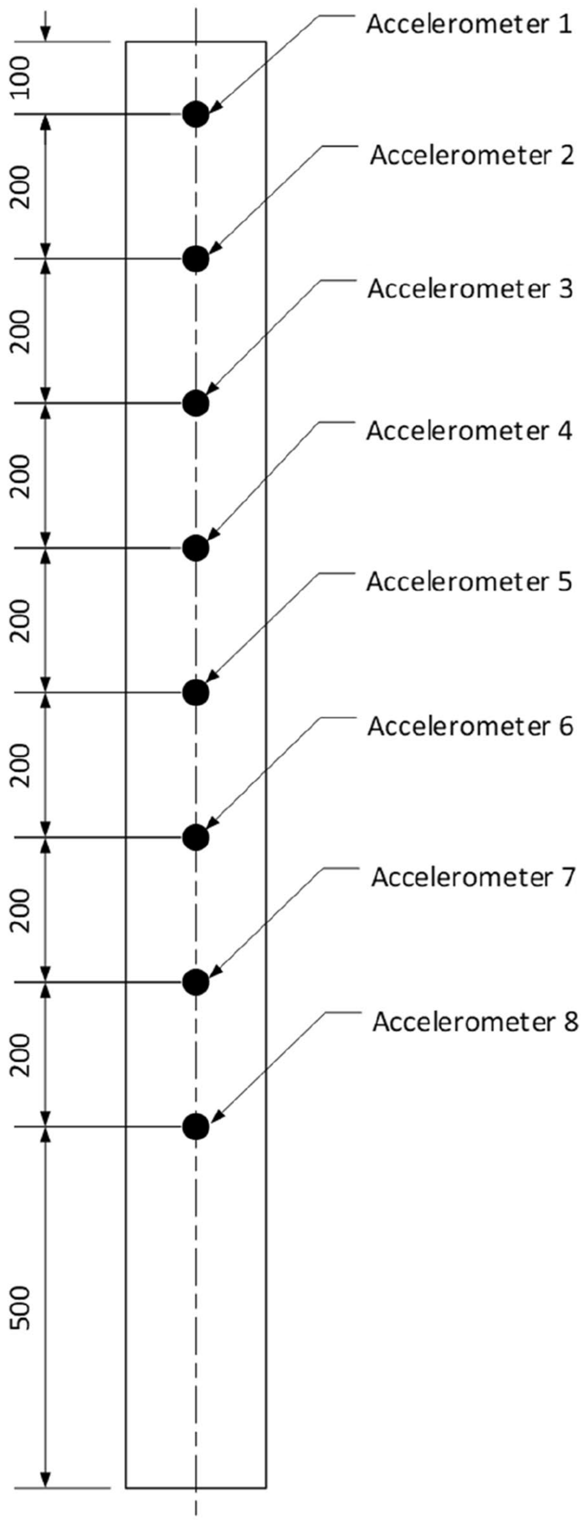

Fig. 5 Accelerometer mounted on the frame – lateral movements

as the mass reduces so will the natural frequency of the structure.

For a time varying mass civil engineering structure like a storage silo, it has been established that there are many factors such as moisture content, wall friction, material lump size, silo geometry, material flow properties, material cohesiveness, filling rate, concentric filling, eccentric filling or radial filling, discharge rate, clamshell gate opening time, eccentricity of the material flow channel during discharge, bin wall liner type, weld bead configuration on bin wall liner, wall roughness, damping and stiffness of the supporting structure that influence the flow of the granular material and generated dynamic loads [19]. Therefore it is expected that Figs. 6, 7 and 8 will change should those parameters vary.

Conditional assessment of structures by analysing the frequency spectrum is quite developed in the field of structural dynamics. Wenzel [20] outlines a number of methods that can be employed to assess the structural integrity of the existing structure. One of these methods is the use of modal testing and analysis [21]. However, modal testing of a time varying mass civil engineering structure such as a silo may require the silo to be put out of operation for safety reasons which can be costly from the operation perspective. It was reported by Tu [16] and recently by Xu and Liang [22] that vibration occurred in all the experiments. As such, the vibration signal captured at each discharge cycle can be used to conditionally assess the structural integrity of the silo without interfering with site operations and throughput resulting in significant savings. However, factors influencing the frequency spectrum needs to be learned before the frequency spectra can be used to conditional monitor the structural integrity of the silo.

Conclusion

It has been demonstrated in this paper that VMD and HT can be used to decompose vibration signals from a time varying mass civil engineering structure such as a silo and generate a frequency spectrum. It is evident that each discharge cycle produces vibrations of varying amplitude and frequency. An inspection of the frequency spectra concludes that there are distinct differences between the frequency spectra recovered from the silo filled with iron ore and the silo filled with sand. Also, it is worth noting that both of these silos are different in construction, the silo containing iron ore was made completely from steel and the silo containing sand was made from polycarbonate tube. These experimental silos are also different in shape. The features decomposed using VMD and HT cements the need to understand the amplitudes and frequencies produced by a engineering structure as it rapidly loses its mass because such frequency

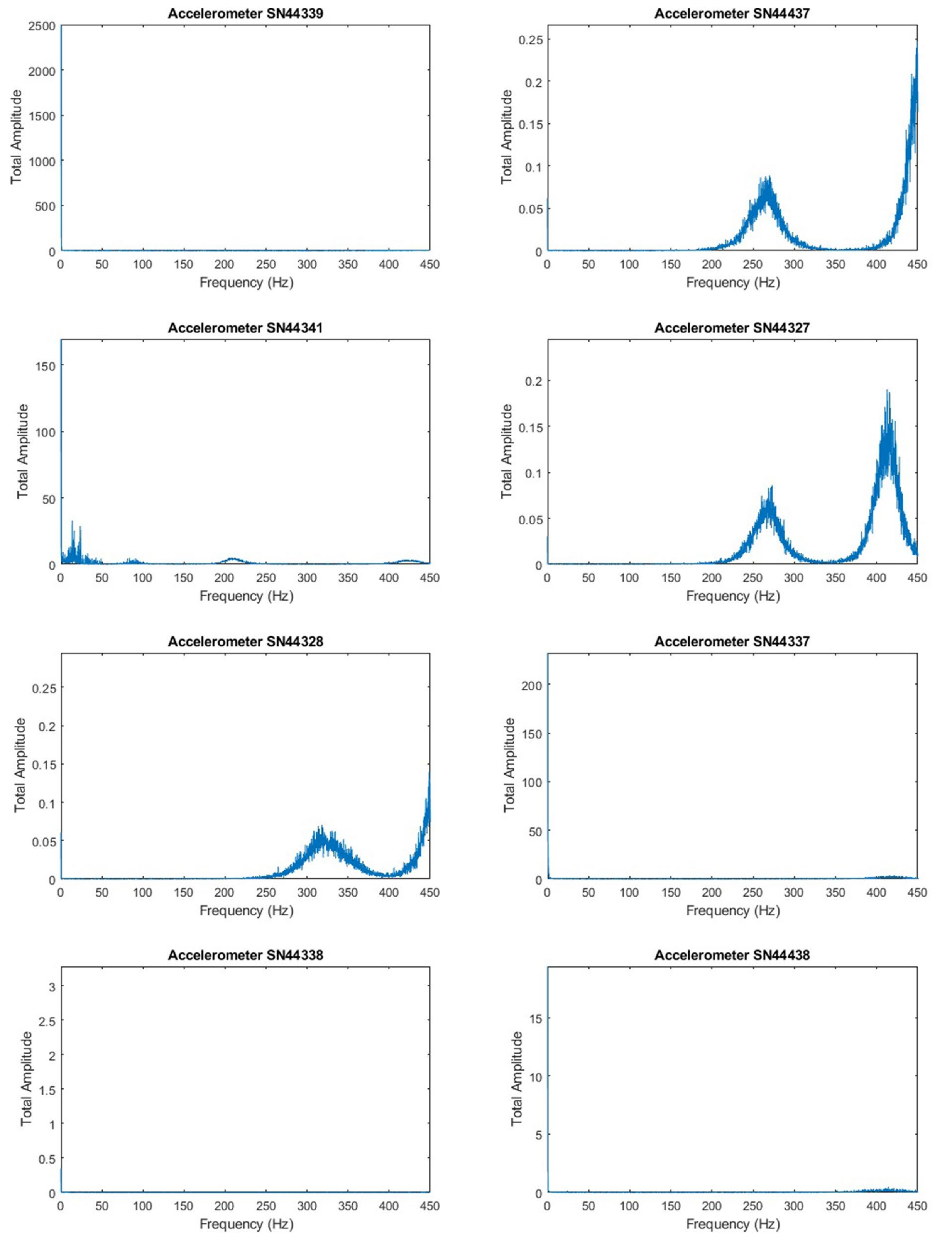

Fig. 6 Frequency spectra of the silo with 300 mm hopper opening – Accelerometer 1 to 8

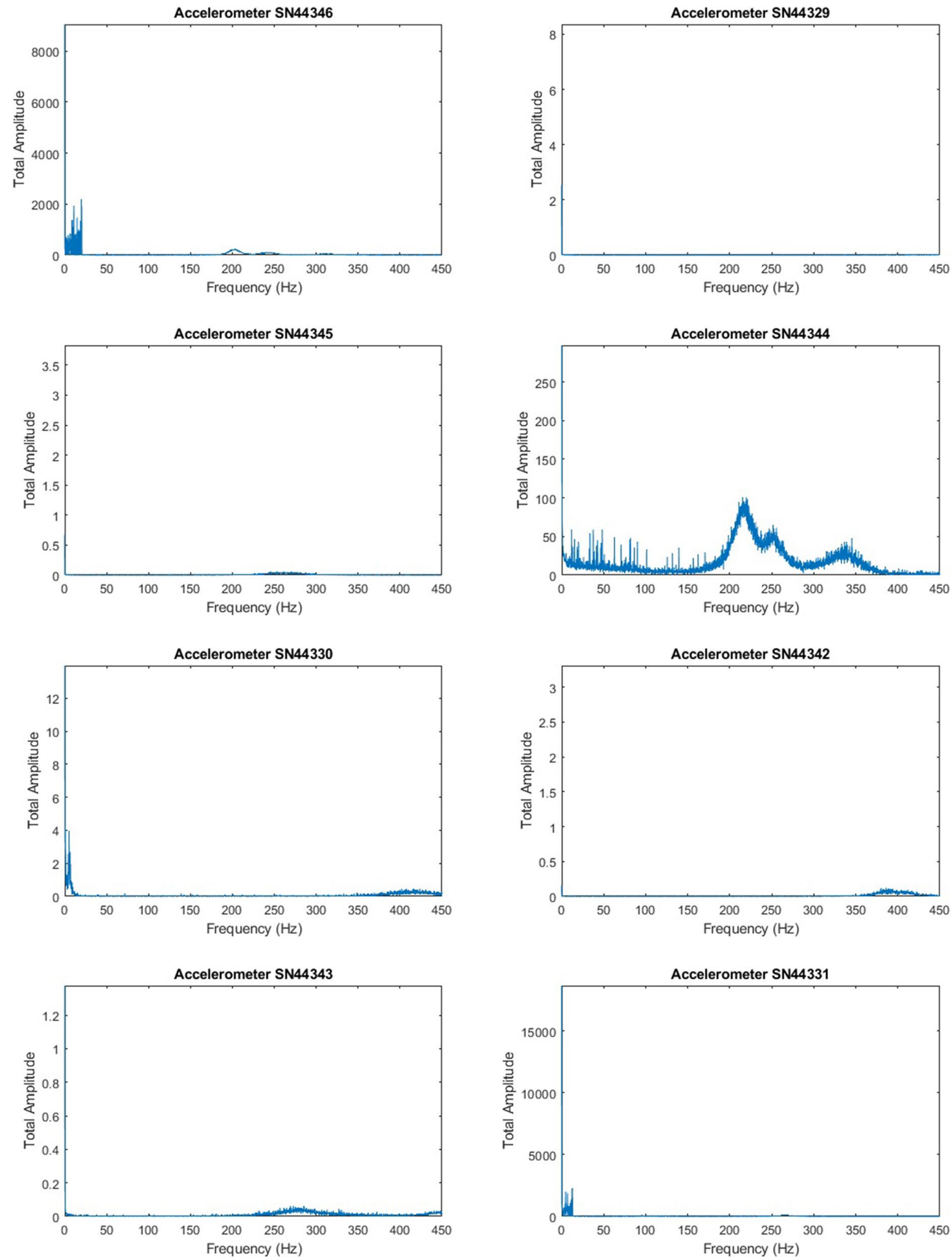

Fig. 7 Frequency spectra of the silo with 300 mm hopper opening – Accelerometer 9 to 16

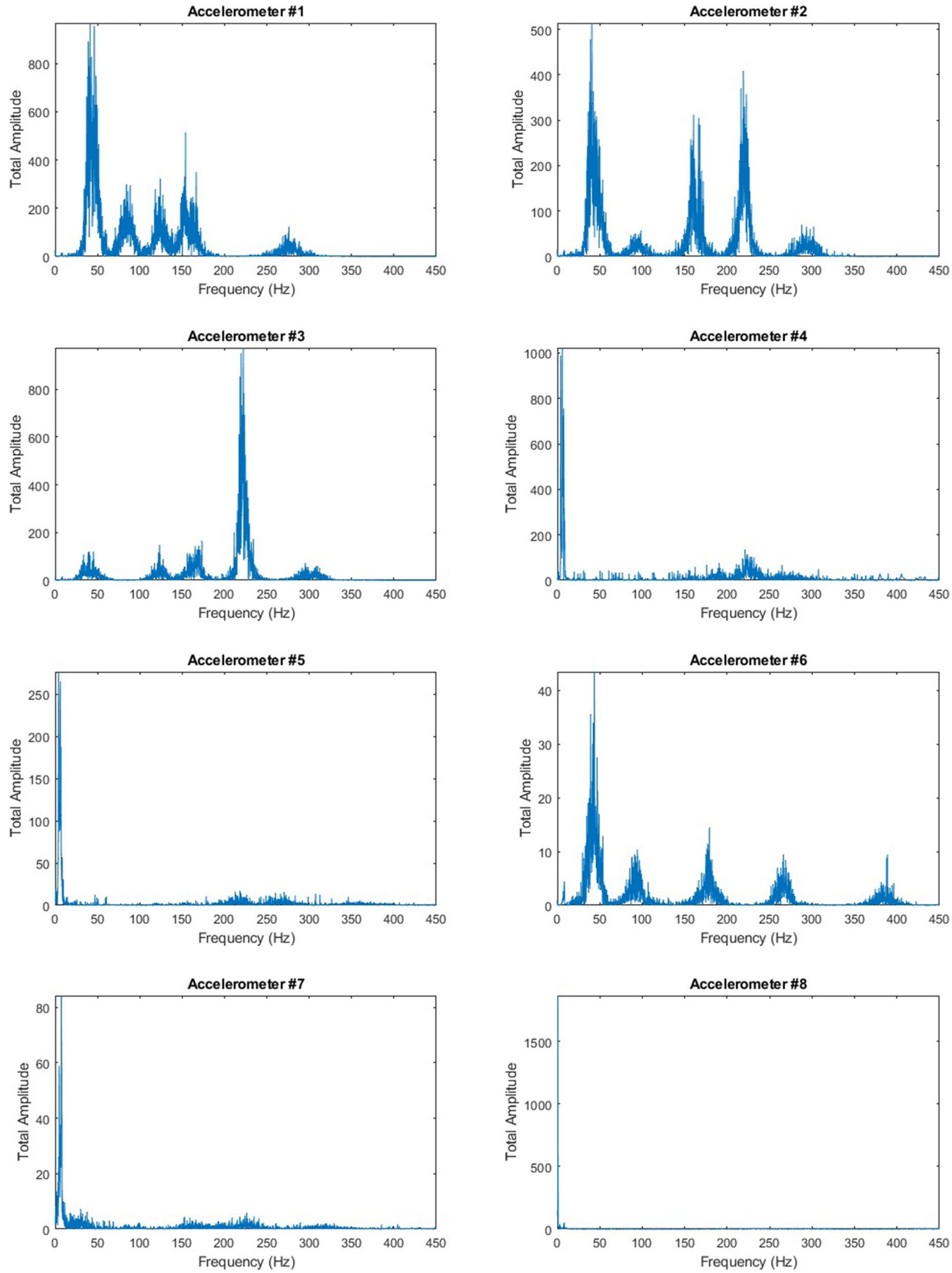

Fig. 8 Frequency spectrum along the height of the polycarbonate tube

spectrum can be developed further to conditionally assess the structural integrity of a time varying mass civil engineering structure in realtime eliminating the need to place the structure out of service for routine inspection. In a case of a silo, factors such as moisture content, wall friction, material lump size, silo geometry, material flow properties, material cohesiveness, filling rate, concentric filling, eccentric filling or radial filling, discharge rate, clamshell gate opening time, eccentricity of the material flow channel during discharge, bin wall liner type, weld bead configuration on bin wall liner, wall roughness, damping and stiffness of the supporting structure need to be studied to determine how such factors influence the frequency spectrum before the method can be employed to conditionally assess the structural integrity of the silo in real time. Furthermore, improvements to methods to decompose nonlinear and nonstationary signals are also required to satisfactorily capture all the features. This area of science is still developing at the time of writing this paper.

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Authors' contributions
Author 1 analysed the data and wrote the manuscript. Author 2 and 3 reviewed the methodologies, data and manuscript. Author 2 supervised the research works undertaken by Author 1 and Griffiths [17]. The author(s) read and approved the final manuscript.

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Availability of data and materials
The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests
The authors declare that they have no competing interests.

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